Permutation and Combinations:

Permutation:

Any arrangement of a set of n objects in a given order is called Permutation of Object. Any arrangement of any $r \leq n$ of these objects in a given order is called an r-permutation or a permutation of n object taken r at a time.

It is denoted by P (n, r)

$$P(n, r) = \frac{n!}{(n-r)!}$$

Theorem: Prove that the number of permutations of n things taken all at a time is n!.

Proof: We know that

$$n_{P_n} = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$$

Example: 4 x $n_{p3}=n+1_{P3}$

Solution: $4 \times \frac{n!}{(n-3)!} = \frac{(n+1)!}{(n+1-3)!}$

$$\frac{4 \text{ x } n!}{(n-3)!} = \frac{(n+1) \text{x } n!}{(n-2)(n-3)!}$$

$4(n-2) = (n+1)$
$4n - 8 = n+1$
$3n = 9$
$n = 3.$

Permutation with Restrictions:

The number of permutations of n different objects taken r at a time in which p particular objects do not occur is

$$n - p_{\,P_r}$$

The number of permutations of n different objects taken r at a time in which p particular objects are present is

$$n - p_{\,P_{r-p}} \text{x } r_{P_p}$$

Example: How many 6-digit numbers can be formed by using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 if every number is to start with '30' with no digit repeated?

Solution: All the numbers begin with '30.'So, we have to choose 4-digits from the remaining 7-digits.

∴ Total number of numbers that begins with '30' is

$7_{P4} = \frac{7!}{(7-4)!} = \frac{7 X 6 X 5 X 4 X 3!}{3!} = 840.$

Permutations with Repeated Objects:

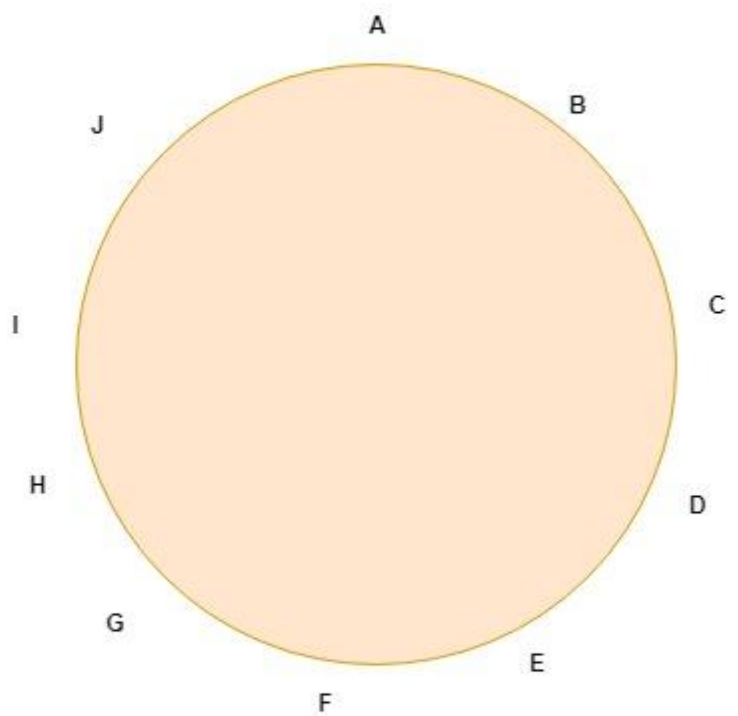
Theorem: Prove that the number of different permutations of n distinct objects taken at a time when every object is allowed to repeat any number of times is given by n^r.

Proof: Assume that with n objects we have to fill r place when repetition of the object is allowed.

Therefore, the number of ways of filling the first place is = n
The number of ways of filling the second place = n
....
....
The number of ways of filling the rth place = n
Thus, the total number of ways of filling r places with n elements is
= n. n. n.............r times $=n^r$.

Circular Permutations:

A permutation which is done around a circle is called Circular Permutation.



Example: In how many ways can get these letters a, b, c, d, e, f, g, h, i, j arranged in a circle?

Solution: (10 - 1) = 9! = 362880

Theorem: Prove that the number of circular permutations of n different objects is (n-1)!

Proof: Let us consider that K be the number of permutations required.

For each such circular permutations of K, there are n corresponding linear permutations. As shown earlier, we start from every object of n object in the circular permutations. Thus, for K circular permutations, we have K...n linear permutations.

Therefore, $K. n = n!$ or $K = \frac{n!}{n}$

$$K = \frac{n \times (n-1)!}{n}$$

$$K = (n-1)!$$

Hence Proved.

Combination:

A Combination is a selection of some or all, objects from a set of given objects, where the order of the objects does not matter. The number of combinations of n objects, taken r at a time represented by n_{C_r} or C (n, r).

$$n_{C_r} = \frac{n!}{r!\,(n-r)!}$$

Proof: The number of permutations of n different things, taken r at a time is given by

$$n_{P_r} = \frac{n!}{(n-r)!}$$

As there is no matter about the order of arrangement of the objects, therefore, to every combination of r things, there are r! arrangements i.e.,

$$n_{P_r} = r!\; n_{C_r} \quad \text{or} \quad n_{C_r} = \frac{n_{P_r}}{r!} = \frac{n!}{(n-r)!r!}, n \geq r$$

Thus,

$$n_{C_r} = \frac{n!}{r!(n-r)!}$$

Example: A farmer purchased 3 cows, 2 pigs, and 4 hens from a man who has 6 cows, 5 pigs, and 8 hens. Find the number m of choices that the farmer has.

The farmer can choose the cows in C (6, 3) ways, the pigs in C (5, 2) ways, and the hens in C (8, 4) ways. Thus the number m of choices follows:

$$m = \binom{6}{3}\binom{5}{4}\binom{8}{4} = \frac{6.5.4}{3.2.1} \text{ x } \frac{5.4}{4} \text{ x } \frac{8.7.6.5}{4.3.2.1} = 20 \text{ x } 10 \text{ x } 70 = 14000$$